U.G. 6th Semester Examination - 2023

MATHEMATICS

[PROGRAMME]

Discipline Specific Elective (DSE)

Course Code : MATH-G-DSE-T-02(A)&(B)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions from selected Option.

OPTION–A

MATH-G-DSE-T-02A

(Linear Programming)

1. Answer any **ten** questions: 2×10=20

   i) Define unbalanced transportation problem.

   ii) Is the following set convex?

   $$S=\{(x, y): x^2+y^2 \le 25\}$$

   iii) When are slack variables used in an L.P.P.?

   iv) State the fundamental theorem of duality.

   v) What is fair game?

vi) When does an alternative solution exist in an L.P.P.?

vii) In a game problem, what is the relation between $\underline{v}$ and $\overline{v}$? ($\underline{v}$, $\overline{v}$ as per convention)

viii) State the condition of unbounded solution in an L.P.P.

ix) Define convex polyhedron.

x) Write the dual of the following L.P.P:

$$\text{Max } Z = CX$$

$$\text{subject to } AX \le b$$

$$X \ge 0.$$

xi) What is the number of occupied cells in a transportation problem with m origins and n destinations, when the solution is non-degenerate?

xii) Write the standard form of an assignment problem.

xiii) What is pure strategy in a game problem?

xiv) What is max-min principle in a game problem?

xv) Which method is more effective to obtain initial solution of a transportation problem?

2. Answer any **four** questions: 5×4=20

i) Solve:

$$\text{Max } Z = 2x_1 + 3x_2$$

$$\text{subject to } x_1 + x_2 \le 8$$

$$x_1 + 2x_2 = 5$$

$$2x_1 + x_2 \le 8$$

$$x_1, x_2 \ge 0.$$

ii) Prove that if the k-th constraint of a primal problem be an equation, then k-th dual variable will be unrestricted in sign.

iii) Prove that a balanced transportation problem always admits a feasible solution.

iv) Solve the following assignment problem:

JOBS

MACHINES	1	2	3	4	5
1	2.5	5	1	5	1
2	2	5	1.5	7	3
3	3	6.5	2	8	3
4	3.5	7	2	9	4.5
5	4	7	3	9	6
6	6	9	5	10	6

v) Solve graphically the game whose pay-off matrix is given below:

		B					
		B_1	B_2	B_3	B_4	B_5	B_6
A	A_1	1	3	−1	4	2	−5
	A_2	−3	5	6	1	2	0

vi) Prove that Hyperplane is a convex set.

3. Answer any **two** questions: 10×2=20

a) i) Solve the following L.P.P. graphically:

$$\text{Max } Z = 4x_1 + 3x_2$$

subject to $x_1 + x_2 \leq 50$

$x_1 + 2x_2 \leq 80$

$2x_1 + x_2 \geq 20$

$x_1, x_2 \geq 0$ 4

ii) Solve the L.P.P. by Simplex method:

$$\text{Minimize } Z = x_1 - 3x_2 + 2x_3$$

subject to $3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2 \geq 0$. 6

b) i) Solve the following transportation problem:

		Destinations				
		D_1	D_2	D_3	D_4	a_i
Origins	O_1	19	30	50	10	7
	O_2	70	30	40	60	9
	O_3	40	08	70	20	18
	b_j	5	8	7	19	

6

ii) Find the dual problem from the following primal problem:

$$\text{Maximize } Z = 3x_1 - 2x_2$$

subject to $x_1 \leq 4$

$x_2 \leq 6$

$x_1 + x_2 = 5$

$-x_2 \leq -1$

$x_1, x_2 \geq 0$ 4

c) i) Find the value of the game and the optimal strategy for each player of the game whose pay-off matrix is given below: 7

		B		
		B_1	B_2	B_3
	A_1	1	−1	−1
A	A_2	−1	−1	3
	A_3	−1	2	−1

ii) Solve the following game problem: 3

		B	
		B_1	B_2
A	A_1	−2	5
	A_2	7	−6

OPTION–B

MATH-G-DSE-T-02B

(Numerical Methods)

1. Answer any **ten** questions: $2\times10=20$

a) Find the absolute error and the relative error in using $\pi = 3.141593$ as an approximation of $\frac{22}{7}$.

b) State the fundamental theorem of the calculus of finite differences.

c) Show that $\Delta^4 y_0 = y_4 - 4y_3 + 6y_2 - 4y_1 + y_0$.

d) Show that $\Delta\{\log f(x)\} = \log\left[1+\frac{\Delta f(x)}{f(x)}\right]$.

e) What do you mean by the degree of precision of a quadrature formula?

f) Prove that $E^{-1} = 1 - \nabla$, where E and ∇ have their usual meanings.

g) If $f(x)=e^{ax+b}$, prove that $f(0)$, $\Delta f(0)$, and $\Delta^2 f(0)$ are in G.P.

h) State the advantages of Lagrange's interpolation.

i) Prove that ∇ is a linear operator.

j) State the basic principle of Newton-Raphson method.

k) What is meant by the diagonally dominant for the system of linear equations?

l) If $f(-2) = f(3) = 7$ and $f(0) = 1$, find $f(10)$.

m) Describe geometrically the convergence of the method of false position.

n) Find $\int_0^2 (x^2 - 4)\,dx$ using Trapezoidal rule by taking two sub-intervals.

o) Use Trapezoidal rule to evaluate $\int_0^6 y(x)\,dx$ for the data:

x	0	1	2	3	4	5	6
y	0.146	0.161	0.167	0.19	0.204	0.217	0.23

p) Apply Runge-Kutta method of fourth order to find an approximate value of $y(0.2)$, given that $\frac{dy}{dx} = x + y$ and $y(0) = 1$.

2. Answer any **four** questions: 5×4=20

a) If $y = 3x^7 - 6x$, find the percentage error in y at $x = 1$ if the error in x is 0.05.

b) Establish Newton's forward interpolation formula.

c) Discuss the method of iteration for numerical solution of an algebraic and transcendental equation.

d) Find a real root of $x^3 + 2x - 6 = 0$ by the method of bisection correct up to two decimal places.

e) Find by suitable interpolation formula, the value of log 4.515 from the following data:

log 4.51 = 0.6542, log 4.52 = 0.6551, log 4.53 = 0.6561, log 4.54 = 0.6571, log 4.55 = 0.6580.

f) Apply Gauss-Seidel iteration method to solve the system of equations:

$$8x - y + z = 18$$

$$x + y - 3z = -6$$

$$2x + 5y - 2z = 3$$

Continue iterations until two successive approximations are identical when rounded to three significant digits.

g) Compute $y(0.2)$ and $y(0.4)$, by Euler's modified method, taking $h = 0.2$, from the differential equation $\frac{dy}{dx} = x + y$, $y(0) = 0$.

3. Answer any **two** questions: 10×2=20

a) i) Establish Lagrange's polynomial interpolation formula.

ii) If $x_1, x_2, ..., x_n$ be the interpolating points and $l_i(x)(i = 0, 1, 2, ..., n)$ be the Lagrangian functions, then show that $\sum_{i=0}^{n} l_i(x) = 1$.

b) i) Using Newton-Raphson method, find the real root of the equation $x^3 - 8x - 4 = 0$ correct up to four significant figures. Give a geometrical significance of the method.

ii) Find the largest eigenvalue and the corresponding eigenvector of the following matrix correct up to four significant figures

$$A = \begin{pmatrix} 9 & 10 & 8 \\ 10 & 5 & -1 \\ 8 & -1 & 3 \end{pmatrix}.$$

c) i) By integrating Newton's forward interpolation formula, obtain the basic form of Simpson's $\frac{1}{3}$rd rule for numerical integration.

ii) Evaluate $\int_0^2 \frac{1}{x^3 + x + 1} dx$ by Simpson's $\frac{1}{3}$rd rule with $h = 0.25$.

d) i) Solve $2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$ by Gauss elimination method.

ii) Given $\frac{dy}{dx} = y^2 + 1$ with $y(0) = 0$, find $f(0.2)$ and $f(0.4)$ by the fourth order Runge-Kutta method, and hence, compare it to the original solution. Here $y = f(x)$.